

Exercise Book: Problem 5

It is given that

$$\forall i, j \in I : [a_i, b_i] \cap [a_j, b_j] \neq \emptyset.$$

A general closed interval $[a, b]$ is defined as $[a, b] = \{x : a \leq x \leq b\}$. From this definition follows that

$$\forall i \in I : a_i \leq b_i \tag{1}$$

and that

$$\forall j \in I : a_j \leq b_j \tag{2}$$

Assume then there would exist a particular $i \in I$ and a particular $j \in I$, say i^* and j^* , such that

$$a_{i^*} > b_{j^*} \tag{3}$$

Under this assumption, (1) implies that $b_{i^*} \geq a_{i^*}$ while (2) implies that $b_{j^*} \geq a_{j^*}$. Joining these two inequalities together by (3) results in

$$b_{i^*} \geq a_{i^*} > b_{j^*} \geq a_{j^*}$$

and hence

$$[a_{i^*}, b_{i^*}] \cap [a_{j^*}, b_{j^*}] = \emptyset$$

which clearly contradicts the assumption that the intersection of any two intervals is nonempty. Thus, the assumption (3) is wrong, and it follows that

$$\forall i, j \in I : a_i \leq b_j \tag{4}$$

Continue then exactly as already shown during the tutorial classes.